Three point SUSY Ward identities without Ghosts

M.L.Walker

ABSTRACT: We utilise a non-local gauge transform which renders the entire action of SUSY QED invariant and respects the SUSY algebra modulo the gauge-fixing condition, to derive two- and three-point ghost-free SUSY Ward identities in SUSY QED. We use the cluster decomposition principle to find the Green's function Ward identities and then takes linear combinations of the latter to derive identities for the proper functions.

Contents

1. Introduction

Supersymmetry (SUSY) is a well-established theory expected to play an essential role in any non-trivial unification of gravity with the gauge forces. More immediate concerns are the solution to the heirachy problem and physics immediately beyond the standard model.

It has long been known that analysis of gauge field theories can be greatly simplified by exploiting the gauge symmetry to derive Ward identities that relate the various Green's functions to each other [1]. The same is even more true of Ward-Takahashi identities, which relate the proper functions of a gauge theory [2].

The application of this concept to SUSY is almost as hold as SUSY itself, with identities relating the propagators of fields within the same multiplet [3]. The application to SUSY gauge theories followed soon with the work of Piquet and Sibold [4] and others [5, 6], who dealt with the complications due to auxiliary fields, Wess-Zumino gauge and gauge fixing, using the BRST quantization to derive SUSY Slavnov-Taylor identities. While these are long established procedures of gauge field theory in the context of gauge fixing, their application to SUSY is not entirely analogous. Ghosts are necessary in non-Abelian gauge theories as part of the gauge-fixing procedure, without which the gauge boson does not have a well-defined propagator. However these considerations do not apply to SUSY. If one is content to simply do perturbative calculations then all propagators are well-defined, once the gauge is fixed, and further ghosts are not necessary.

This is not to say that such methods are not valuable. Indeed, they have allowed the development of renormalisation schemes that respect both SUSY and gauge symmetry [7, 8, 9]. Still, the ghosts are not inherent to SUSY in the way that they are to gauge theory, even in Wess-Zumino gauge. Furthermore, this now standard technique requires the introduction of ghosts even to the Abelian SUSY QED [6], where there were none in the non-SUSY case.

The complications related to the gauge-fixing term in deriving SUSY identities can be traced back to the use of the transforms derived by Wess and Zumino [10]. When working in the Wess-Zumino gauge, in which the gauge dependant superpartners of the photon have been gauged away, their transform leaves the gauge invariant part of the SUSY QED action invariant but spoils the $U(1)$ gauge fixing condition. It is at this point that standard treatments turn to the BRST formalism. We demonstrated an alternative approach in [11]. Our approach was to treat the breaking of the $U(1)$ gauge condition in the same way that Wess and Zumino treated the breaking of the Wess-Zumino (WZ) gauge, by introducing another gauge transformation to restore it. The resulting transform had the attractive feature of obeying the SUSY algebra modulo the gauge condition. We went on in that paper to find SUSY Ward identities relating the propagators not only of the matter multiplet, but also of the gauge multiplet. These identities all held exactly at tree level, something that had not been achieved with the gauge multiplet before without using ghost fields. Using the cluster decomposition principle [12, 13] to handle the non-linearities, as demonstrated by [6], we then went on to find an identity relating some three-point Green's functions.

Another important incentive for introducing ghosts is that the Wess-Zumino transforms, as well as our own, are non-linear. Indeed, the additional correction found in [11] is also non-local. This forbids the use of the Legendre transform or the effective action in any way, unless the transform is linearised as the BRST transforms are. We shall circumvent this problem by taking linear combinations of the Green's function Ward identities.

In this paper we continue our previous work, begining with a brief summary of our transform and its properties in section 2. In section 3 we give a detailed discussion of identities relating three-point Green's functions and present them in their entirety in appendix A. In section 4 we tackle the problem of SUSY Ward identities relating the vertices. Effective action techniques cannot be used for reasons already noted. The vertex identities themselves are given in appendix C. It shall transpire that the vertex Ward identities have already been found by an independant method [14], and that their solution, when subject to reasonable constraints like gauge invariance, has already been found in its most general form which we present in appendix D.

2. The Wess-Zumino-Lorentz Transform

Since the transformations used in this paper are newly-discovered [11], we briefly restate them here. The basic problem is to modify the identities found by Wess and Zumino to leave the gauge invariant part of the action invariant [10] so that they also leave the gauge-fixing component invariant. Their transforms are

$$\delta_{WZ} a = -i \bar{\zeta} \psi$$

$$\delta_{WZ}b = \bar{\zeta}\gamma_5\psi$$

$$\delta_{WZ}\psi = (f + i\gamma_5 g)\zeta + i\,\slashed{\partial}(a + i\gamma_5 b)\zeta - e\,\slashed{A}(a - i\gamma_5 b)\zeta$$

$$\delta_{WZ}f = \bar{\zeta}\,\slashed{\partial}\psi - e\bar{\zeta}[a\lambda + ib\gamma_5\lambda - i\,\slashed{A}\psi]$$

$$\delta_{WZ}g = i\bar{\zeta}\gamma_5\,\slashed{\partial}\psi - ei\bar{\zeta}[a\gamma_5\lambda + ib\lambda + i\,\slashed{A}\gamma_5\psi], \tag{2.1}$$

$$\delta_{WZ}A_\mu = \bar{\zeta}\gamma_\mu\lambda - i\bar{\zeta}\partial_\mu\chi$$

$$\delta_{WZ}\lambda = \frac{1}{2}(\gamma^\nu\gamma^\mu - \gamma^\mu\gamma^\nu)\partial_\mu A_\nu\zeta + i\gamma_5 D\zeta$$

$$\delta_{WZ}D = i\bar{\zeta}\gamma_5\,\slashed{\partial}\lambda. \tag{2.2}$$

The effect of δ_{WZ} on the Lorentz gauge condition is

$$\delta_{WZ}(\partial \cdot A) = \bar{\zeta}\,\slashed{\partial}\lambda, \tag{2.3}$$

but this can be undone with the gauge transformation

$$\delta_L A_\mu = -\partial_\mu\frac{\slashed{\partial}}{\Box}\lambda, \tag{2.4}$$

so that $(\delta_{WZ} + \delta_L)\partial \cdot A = 0$. The effect of δ_L on all the fields in the theory is

$$\delta_L a = i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda a,$$

$$\delta_L b = i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda b,$$

$$\delta_L\psi = i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda\psi,$$

$$\delta_L f = i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda f,$$

$$\delta_L g = i\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda g,$$

$$\delta_L A_\mu = \partial_\mu\bar{\zeta}\frac{\slashed{\partial}}{\Box}\lambda,$$

$$\delta_L\lambda = 0,$$

$$\delta_L D = 0, \tag{2.5}$$

Since δ_L is a gauge transformation it leaves the gauge-invariant part of the action unchanged, so the complete SUSY transform of SUSY QED in component notation is

$$\delta_{WZL} = \delta_{WZ} + \delta_L. \tag{2.6}$$

This complete transform derives simple and useful identities among the propagators of SUSY QED [11]. Most notable is the identity relating the photon and photino propagators, found thus:

$$0 = \langle\delta_{WZL}(A_\mu(x)\lambda(y))\rangle$$

$$= \langle A_\mu(x)A_\beta(y)\rangle_y\partial_\alpha\sigma^{\beta\alpha}\zeta - \langle\lambda(y)\bar{\lambda}(x)\rangle\gamma_\mu\zeta + \langle\lambda(y)\bar{\lambda}(x)\rangle\frac{x\partial_\mu}{x\slashed{\partial}}\zeta, \tag{2.7}$$

After converting to momentum space, the surviving members of the gauge multiplet have the following propagators in WZ gauge:

$$\langle A_\mu A_\nu \rangle(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{1+\Pi(k)} + \xi\frac{k_\mu k_\nu}{k^4}, \tag{2.8}$$

$$\langle \lambda\bar\lambda \rangle(k) = \frac{-i}{A_\lambda(k)\,\slashed{k}}, \tag{2.9}$$

$$\langle DD \rangle(k) = \frac{1}{A_\lambda(k)}. \tag{2.10}$$

where

$$A_\lambda(k) = 1 + \Pi(k), \tag{2.11}$$

This relationship between the photon and photino propagators is what one would naively expect [15].

We shall also need the identities relating the electron and selectron propagators. One might expect that since their wavefunction renormalisation is dependant on the gauge parameter ξ, the SUSY violation of the gauge-fixing term would cause the electron and selectron wavefunction renormalisations to differ, at least nonperturbatively. However our transformations are not violated by covariant gauge-fixing so such reasoning does not apply. The Ward identity relating the ψ and a propagators is

$$\begin{aligned}
0 &= \langle \delta_{WZL}(\bar\psi(x)a(y)) \rangle \\
&= -i\bar\zeta\langle \psi(y)\bar\psi(x) \rangle + \bar\zeta\langle a(y)f^*(x) \rangle - i\bar\zeta\,_x\slashed{\partial}\langle a(y)a^*(x) \rangle,
\end{aligned} \tag{2.12}$$

as found originally in [3]. The non-linear contribution to this and all propagator Ward identities vanishes by the cluster decomposition principle [6, 12, 13]. Taking the most general form for the electron propagator

$$\langle \psi\bar\psi \rangle(p) = \frac{i}{\slashed{p}A(p^2) + B(p^2)}, \tag{2.13}$$

we then have

$$\langle aa^* \rangle(p^2) = \frac{A(p^2)}{p^2 A(p^2)^2 - B(p^2)^2}, \tag{2.14}$$

$$\langle af^* \rangle(p^2) = \langle fa^* \rangle(p^2) = \frac{B(p^2)}{p^2 A(p^2)^2 - B(p^2)^2}, \tag{2.15}$$

$$\langle ff^* \rangle(p^2) = \frac{p^2 A(p^2)}{p^2 A(p^2)^2 - B(p^2)^2}, \tag{2.16}$$

where the last two propagators are given by [3]

$$\begin{aligned}
0 &= \langle \delta_{WZL}(\bar\psi(x)f(y)) \rangle \\
&= \bar\zeta_y\,\slashed{\partial}\langle \psi(y)\bar\psi(x) \rangle + \bar\zeta\langle f(y)f^*(x) \rangle - i\bar\zeta_x\,\slashed{\partial}\langle f(y)a^*(x) \rangle.
\end{aligned} \tag{2.17}$$

Similarly,

$$\langle bb^* \rangle(p^2) = \frac{A(p^2)}{p^2 A(p^2)^2 - B(p^2)^2}, \tag{2.18}$$

$$\langle bg^* \rangle(p^2) = \langle gb^* \rangle(p^2) = \frac{B(p^2)}{p^2 A(p^2)^2 - B(p^2)^2}, \tag{2.19}$$

$$\langle gg^* \rangle(p^2) = \frac{p^2 A(p^2)}{p^2 A(p^2)^2 - B(p^2)^2}. \tag{2.20}$$

We shall use these equations in sec. 4.

3. Green's Function Ward Identities

Having described our SUSY tranform and used it to restrict the form of the propagators, we now apply the same procedure to the three-point Green's functions. Knowing the SUSY WIs among these will no doubt be an asset to those computing S-matrices. As already mentioned, SUSY ST identities for these functions are already known [4, 5, 6] but we are the first to use completely invariant WIs in SUSY QED.

We begin with an illustrative example. Consider

$$
\begin{aligned}
0 &= \langle \delta_{WZL}(a(x) A_\mu(z) \bar\psi(y)) \rangle \\
&= i\bar\zeta \langle \bar\psi(y) A_\mu(z) \psi(x) \rangle - i\bar\zeta \langle (\slashed{\partial} a^*(y)) A_\mu(z) a(x) \rangle + \bar\zeta \langle f^*(y) A_\mu(z) a(x) \rangle \\
&\quad + \bar\zeta \langle a(x) A_\mu(z) (a^* \slashed{A})(y) \rangle - \bar\zeta \gamma_\mu \langle \bar\psi(y) \lambda(z) a(x) \rangle + \bar\zeta \langle \bar\psi(y) \left(\partial_\mu \frac{\slashed{\partial}}{\Box} \lambda(z) \right) a(x) \rangle. \tag{3.1}
\end{aligned}
$$

These are all the non-vanishing terms that arise from our transform, eq. (2.6). The term $\langle a(x) A_\mu(z) f^*(y) \rangle$ does not vanish but is easily calculated using the $\langle a f^* \rangle$ propagator (eq. (2.15)) discussed in the last section. They are identical to those generated by the Wess-Zumino transforms [6, 10] except for the last term.

The third-last term contains a non-linear contribution whose evaluation utilises the cluster decomposition principle [6, 12]. A convenient procedure is to replace that Green's function with products of its sub-graphs that have a^* and A_μ in separate connected components. This gives us

$$\bar\zeta \langle a(x) A_\mu(z) (a^* \slashed{A})(y) \rangle = \langle a(x) a^*(y) \rangle_C \gamma^\nu \langle A_\nu(y) A_\mu(z) \rangle_C. \tag{3.2}$$

substituting this and converting all Green's functions to connected Green's functions gives

$$
\begin{aligned}
0 &= -i \langle \bar\psi(y) A_\mu(z) \psi(x) \rangle_C + i \langle (\slashed{\partial} a^*(y)) A_\mu(z) a(x) \rangle_C - \langle f^*(y) A_\mu(z) a(x) \rangle_C \\
&\quad - \langle a(x) a^*(y) \rangle_C \gamma^\nu \langle A_\nu(y) A_\mu(z) \rangle_C + \gamma_\mu \langle \bar\psi(y) \lambda(z) a(x) \rangle_C \\
&\quad - \langle \bar\psi(y) \left(\partial_\mu \frac{\slashed{\partial}}{\Box} \lambda(z) \right) a(x) \rangle_C. \tag{3.3}
\end{aligned}
$$

The sign changes occur because a transition from general to connected Greens' functions causes a sign change of $(-1)^N$ for N-point functions in Minkowski space [16]. It is a quick

and easy task to check that eq. (3.3) holds at the bare level, something that previous calculations could not achieve without the introduction of ghost fields.

Another helpful example is

$$
\begin{aligned}
0 &= \langle \delta_{WZL}(a\lambda a^*)\rangle \\
&= -C\langle a^*(y)\bar{\lambda}(z)\psi(x)\rangle_C^T C^{-1}\zeta - \langle\bar{\psi}(y)\lambda(z)a(x)\rangle_C\zeta + i\langle a^*(y)(_z\partial_\mu A_\nu(z))a(x)\rangle_C\sigma^{\nu\mu}\zeta \\
&\quad - \langle\lambda(z)(\frac{\not{\partial}_x}{_x\Box}\bar{\lambda}(x)))\rangle_C\langle a(x)a^*(y)\rangle_C\zeta + \langle a(x)a^*(y)\rangle_C\langle\lambda\left(\bar{\lambda}(y)\frac{\not{\partial}}{\Box}\right)\rangle_C\zeta.
\end{aligned}
\tag{3.4}
$$

Again, these are the same terms that one would expect from the WZ transforms, except for the last two which come from δ_L, and again the identity vanishes simply and easily at bare level without introducing additional ghost fields.

$\langle\delta_{WZL}(\ldots)\rangle = 0$	SWI	$\langle\delta_{WZL}(\ldots)\rangle = 0$	SWI
$\langle\delta_{WZL}(a(p)a^*(q)\lambda(p-q))\rangle$	A.1	$\langle\delta_{WZL}(a(p)D(p-q)\bar{\psi}(q))\rangle$	A.14
$\langle\delta_{WZL}(b(p)b^*(q)\lambda(p-q))\rangle$	A.2	$\langle\delta_{WZL}(b(p)D(p-q)\bar{\psi}(q))\rangle$	A.15
$\langle\delta_{WZL}(f(p)a^*(q)\lambda(p-q))\rangle$	A.3	$\langle\delta_{WZL}(f(p)D(p-q)\bar{\psi}(q))\rangle$	A.16
$\langle\delta_{WZL}(g(p)b^*(q)\lambda(p-q))\rangle$	A.4	$\langle\delta_{WZL}(\psi(p)D(p-q)g^*(q))\rangle$	A.17
$\langle\delta_{WZL}(a(p)f^*(q)\lambda(p-q))\rangle$	A.5	$\langle\delta_{WZL}(b(p)\lambda(p-q)a^*(q))\rangle$	A.18
$\langle\delta_{WZL}(b(p)g^*(q)\lambda(p-q))\rangle$	A.6	$\langle\delta_{WZL}(a(p)\lambda(p-q)b^*(q))\rangle$	A.19
$\langle\delta_{WZL}(f(p)f^*(q)\lambda(p-q))\rangle$	A.7	$\langle\delta_{WZL}(g(p)\bar{\lambda}(p-q)a^*(q))\rangle$	A.20
$\langle\delta_{WZL}(g(p)g^*(q)\lambda(p-q))\rangle$	A.8	$\langle\delta_{WZL}(f(p)\bar{\lambda}(p-q)b^*(q))\rangle$	A.21
$\langle\delta_{WZL}(f(p)A_\mu(p-q)\bar{\psi}(q))\rangle$	A.9	$\langle\delta_{WZL}(a(p)\bar{\lambda}(p-q)g^*(q))\rangle$	A.22
$\langle\delta_{WZL}(g(p)A_\mu(p-q)\bar{\psi}(q))\rangle$	A.10	$\langle\delta_{WZL}(b(p)\bar{\lambda}(p-q)f^*(q))\rangle$	A.23
$\langle\delta_{WZL}(a(p)A_\mu(p-q)\bar{\psi}(q))\rangle$	A.11	$\langle\delta_{WZL}(g(p)\bar{\lambda}(p-q)f^*(q))\rangle$	A.24
$\langle\delta_{WZL}(b(p)A_\mu(p-q)\bar{\psi}(q))\rangle$	A.12	$\langle\delta_{WZL}(f(p)\bar{\lambda}(p-q)g^*(q))\rangle$	A.25
		$\langle\delta_{WZL}(\psi_\alpha(q)\bar{\psi}^\beta(p)\bar{\lambda}^\kappa(p-q))\rangle$	A.13

Table 1: Each SWI is derived from the invariance of the action to the transform δ_{WZL}. The starting point of each SWI (indicated by its equation number) is given in this table.

We give the complete set of identities in appendix A modulo duplications due to charge conjugation. Table 1 lists all the possible starting points for deriving identities, together with their corresponding equation number.

The lesson of this section is unmistakable. The three-point Green's functions of SUSY QED are related by very simple SUSY WIs that do not require ghost fields or STI. Conventional approaches are an adaptation of the WZ transforms which were derived only for the gauge invariant part of SUSY QED.

4. Vertex Ward Identities

We have demonstrated the usefulness of the WZL transform for S-matrix calculations, but what about renormalisation? In this case we seem to be stuck because our transform is both non-local and non-linear, forbidding any calculation that requires the Legendre transformation in any form. The conventional method of dealing with this problem is to introduce ghost fields that couple to the non-linear terms. While this method is valid we would like, having come this far, to avoid the introduction of unphysical fields. A little inspection reveals that the propagator WIs (eqs. (2.13)-(2.20)) allow the propagator denominators to be factored out. Taking linear combinations produces a comprehensive set of identities relating the vertices of SUSY QED to each other. We demonstrate by considering eqs. (A.9,A.11). We transform to momentum space, and take the combination

$$A(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)f(p))\rangle - B(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)a(p))\rangle, \tag{4.1}$$

.

The next step is to expand each three-point Green's function in terms of its propagators and vertex. For example, the electron-photon Green's function expands to

$$\langle\bar{\psi}(q)A_\mu(p-q)\psi(p)\rangle_C = \langle\psi\bar{\psi}\rangle(p)\Gamma^\nu_{\bar{\psi}A\psi}(p,q)\langle\psi\bar{\psi}\rangle(q)\langle A_\nu A_\mu\rangle(p-q). \tag{4.2}$$

Functions with scalar legs are a little more complicated. For example

$$\langle\bar{\psi}(q)\lambda(p-q)a(p)\rangle_C = \langle\lambda\bar{\lambda}\rangle(p-q)(\Gamma_{a^*\bar{\lambda}\psi}(p,q)\langle aa^*\rangle(p)+\Gamma_{f^*\bar{\lambda}\psi}(p,q)\langle af^*\rangle(p))\langle\psi\bar{\psi}\rangle(q), \tag{4.3}$$

and

$$\langle\bar{\psi}(q)\lambda(p-q)f(p)\rangle_C = \langle\lambda\bar{\lambda}\rangle(p-q)(\Gamma_{a^*\bar{\lambda}\psi}(p,q)\langle fa^*\rangle(p)+\Gamma_{f^*\bar{\lambda}\psi}(p,q)\langle ff^*\rangle(p))\langle\psi\bar{\psi}\rangle(q). \tag{4.4}$$

Substituting in this way produces the identity

$$\begin{aligned}
i\sigma^{\mu\nu}&(p-q)_\nu\Gamma_{\bar{\lambda}f^*\psi}(p,q)\\
&= \Gamma^\mu_{\bar{\psi}A\psi}(p,q) - i\not{q}\Gamma^\mu_{f^*Af}(p,q) + i\Gamma^\mu_{f^*Aa}(p,q) - ie\gamma^\mu A(p^2),
\end{aligned} \tag{4.5}$$

after dividing out common factors. Similarly, the combination

$$B(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)a(p))\rangle - p^2A(p^2)\langle\delta(\bar{\psi}(q^2)A_\mu(p-q)f(p))\rangle, \tag{4.6}$$

gives the vertex WI

$$\begin{aligned}
i\sigma^{\mu\nu}&(p-q)_\nu\Gamma_{\bar{\lambda}a^*\psi}(p,q)\\
&= i\Gamma^\mu_{a^*Aa}(p,q) - i\not{q}\Gamma^\mu_{a^*Af}(p,q) - e\gamma^\mu\langle\psi\bar{\psi}\rangle^{-1}(q) - \not{p}\Gamma^\mu_{\bar{\psi}A\psi}(p,q)\\
&\quad +ie\gamma^\mu B(p^2),
\end{aligned} \tag{4.7}$$

$$i\sigma^{\mu\nu}(p-q)_\nu\Gamma_{\bar{\lambda}b^*\psi}(p,q) \tag{4.8}$$

These are the only identities produced in Feynman gauge ($\xi = 1$), In any other gauge they each produce a second identity because the terms proportional to ($\xi-1$) must cancel among

Combination of Green's function Ward identities	Vertex SWI
$q^2 A(q^2)(p^2 A(p^2)\text{(A.1)} - B(p^2)\text{(A.3)}) - B(q^2)(p^2 A(p^2)\text{(A.5)} - B(p^2)\text{(A.7)})$	C.1
$q^2 A(q^2)(p^2 A(p^2)\text{(A.2)} - B(p^2)\text{(A.4)}) - B(q^2)(p^2 A(p^2)\text{(A.6)} - B(p^2)\text{(A.8)})$	C.2
$q^2 A(q^2)(A(p^2)\text{(A.3)} - B(p^2)\text{(A.1)}) - B(q^2)(A(p^2)\text{A.7} - B(p^2)\text{(A.5)})$	C.3
$q^2 A(q^2)(A(p^2)\text{(A.4)} - B(p^2)\text{(A.2)}) - B(q^2)(A(p^2)\text{(A.8)} - B(p^2)\text{(A.6)})$	C.4
$A(q^2)(A(p^2)\text{(A.7)} - B(p^2)\text{(A.5)}) - B(q^2)(A(p^2)\text{(A.3)} - B(p^2)\text{(A.1)})$	C.5
$A(q^2)(A(p^2)\text{(A.8)} - B(p^2)\text{(A.6)}) - B(q^2)(A(p^2)\text{(A.4)} - B(p^2)\text{(A.2)})$	C.6
$A(p^2)\text{(A.9)} - B(p^2)\text{(A.11)}$	C.7
$A(p^2)\text{(A.10)} - B(p^2)\text{(A.12)}$	C.8
$p^2 A(p^2)\text{(A.11)} - B(p^2)\text{(A.9)}$	C.9
$p^2 A(p^2)\text{(A.12)} - B(p^2)\text{(A.10)}$	C.10
A.13	C.11
$p^2 A(p^2)\text{(A.14)} - B(p^2)\text{(A.16)}$	C.14
$p^2 A(p^2)\text{(A.15)} - B(p^2)\text{(A.17)}$	C.15
$A(p^2)\text{(A.16)} - B(p^2)\text{(A.14)}(\delta\psi(y)\delta D(z)\delta f^*(p))$	C.16
$A(p^2)\text{(A.17)} - B(p^2)\text{(A.15)}$	C.17
$A(q^2)(p^2 A(p^2)\text{(A.19)} - B(p^2)\text{(A.21)}) - B(q^2)(p^2 A(p^2)\text{(A.22)} - B(p^2)\text{(A.25)})$	C.18
$A(q^2)(p^2 A(p^2)\text{(A.18)} - B(p^2)\text{(A.20)}) - B(q^2)(p^2 A(p^2)\text{(A.23)} - B(p^2)\text{(A.24)})$	C.19
$q^2 A(q^2)(p^2 A(p^2)\text{(A.22)} - B(p^2)\text{(A.25)}) - B(q^2)(p^2 A(p^2)\text{(A.19)} - B(p^2)\text{(A.21)})$	C.20
$A(q^2)(p^2 A(p^2)\text{(A.23)} - B(p^2)\text{(A.24)}) - B(q^2)(p^2 A(p^2)\text{(A.18)} - B(p^2)\text{(A.20)})$	C.21
$q^2 A(q^2)(A(p^2)\text{(A.20)} - B(p^2)\text{(A.18)}) - B(q^2)(A(p^2)\text{(A.24)} - B(p^2)\text{(A.23)})$	C.22
$q^2 A(q^2)(A(p^2)\text{(A.21)} - B(p^2)\text{(A.19)}) - B(q^2)(A(p^2)\text{(A.25)} - B(p^2)\text{(A.22)})$	C.23
$q^2 A(q^2)(A(p^2)\text{(A.25)} - B(p^2)\text{(A.22)}) - B(q^2)(A(p^2)\text{(A.21)} - B(p^2)\text{(A.19)})$	C.24
$A(q^2)(A(p^2)\text{(A.24)} - B(p^2)\text{(A.23)}) - B(q^2)(A(p^2)\text{(A.20)} - B(p^2)\text{(A.18)})$	C.25

Table 2: The SWIs relating the vertices are found by taking linear combinations of the Green's function SWIs. The corresponding combinations for each vertex SWI (indicated by its equation number) is given in this table.

themselves due to gauge invariance. Since this second identity is not found in Feynman gauge it must not contain any additional information. This expection is indeed fulfilled and it is straightforward to check that it is made redundant by the Ward-Takahashi identities (see appendix B) and the two-point SUSY WIs (2.13-2.16).

Appendix C lists all the SUSY vertex WIs. Table 2 lists which combination of Green's

function WIs corresponds to which vertex identity.

The derivation for some identities utilizes charge invariance, which imposes

$$C\Gamma^{\mu}_{x^{\dagger}Ay}(p,q)^{T}C^{-1} = -\Gamma^{\mu}_{y^{\dagger}Ax}(-q,-p),$$
$$C\Gamma_{x^{\dagger}Dy}(p,q)^{T}C^{-1} = -\Gamma_{y^{\dagger}Dx}(-q,-p), \tag{4.9}$$

where x, y are members of the chiral multiplet.

The result of this section is quite unexpected as, to our knowledge, such identities have only been derived previously using the effective action. Indeed, one might quite reasonably ask how this approach went unnoticed for so long. The answer is that Ward identities were discovered in the context of gauge symmetries where this factorisation approach simply doesn't work. For a quick example, try to derive the original Ward-Takahashi identity relating the electron-photon vertex to the electron propagator, *ie.*

$$0 = \langle \delta_G(\bar{\psi}(y)A_{\mu}(z)\psi(x)) \rangle.$$

The result is useless, $0 = 0$.

We finish this section by observing that the vertex Ward identities derived here are identical to those derived by a different method. In that work [14] we observed that the effective action could be left in superfield form until after the Legendre transform, and then the WZ gauge adopted. While a Legendre transform of non-linear terms was not committed, one might still hold reservations about such an approach, which in any event cannot be applied to non-Abelian theories. By deriving the same identities in an independant way that does not use the effective action or Legendre transform, we vindicate that approach. We are also permitted to profit from that earlier work which went on to find the most general solution for the vertices, given reasonable assumptions such as charge conjugation invariance. We include it in this paper for convenience but relegate it to appendix D.

5. Discussion

Taking the SUSY transformation that leaves the entire SUSY QED action completely invariant, we have derived SUSY WIs relating the three-point Green's functions, which are listed in appendix A. As was the case with propagators [11], these identities hold at bare level without the introduction of ghost fields, in contrast to other approaches [5, 6]. We then found that by utilising the SUSY propagator WIs, we could take linear commbinations of our Green's function identities which revealed vertex WIs after dividing out common factors, an approach that does not work with gauge symmetry. These identities are listed in appendix C. Notably, the resulting identities are identical to those derived previously by adopting the WZ gauge after moving to the effective action, and whose most general solution is already known [14]. We have restated this solution in appendix D.

This paper was limited to the Abelian SUSY gauge theory. The application of our approach to non-Abelian theories, which are clearly of greater importance, has yet to be demonstrated. This is a topic for future work.

A. Green's function Ward identities

This appendix lists all the SUSY Green's function Ward identities of SUSY QED modulo charge conjugation. The starting points for deriving them are given in table 1. Note that all Green's functions shown here are assumed to be connected, although the subscript $_C$ has been neglected for the sake of clarity.

$$
\begin{aligned}
0 = & \ C\langle a^*(q)\bar{\lambda}(p-q)\psi(p)\rangle^T C^{-1} + \langle\bar{\psi}(q)\lambda(p-q)a(p)\rangle \\
& -(p-q)_\mu\langle a^*(q)A_\nu(p-q)a(p)\rangle\sigma^{\nu\mu} + ie\frac{(\not{p}-\not{q})}{(p-q)^2}\langle\lambda\bar{\lambda}\rangle(p-q)\langle aa^*\rangle(q) \\
& -ie\langle aa^*\rangle(p)\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}.
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
0 = & \ -C\langle b^*(q)\bar{\lambda}(p-q)\psi(p)\rangle^T C^{-1}\gamma_5 - \langle\bar{\psi}(q)\lambda(p-q)b(p)\rangle\gamma_5 \\
& -i(p-q)_\mu\langle b^*(q)A_\nu(p-q)b(p)\rangle\sigma^{\nu\mu} + e\langle\lambda\bar{\lambda}\rangle(p-q)\frac{\not{p}-\not{q}}{(p-q)^2}\langle bb^*\rangle(p) \\
& -e\langle bb^*\rangle(q)\langle\lambda\bar{\lambda}\rangle(p-q)\frac{\not{p}-\not{q}}{(p-q)^2}.
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
0 = & \ iC\langle a^*(q)\bar{\lambda}(p-q)\psi(p)\rangle^T C^{-1} \not{p} + i\langle\bar{\psi}(q)\lambda(p-q)f(p)\rangle \\
& -i(p-q)_\mu\langle a^*(q)A_\nu(p-q)f(p)\rangle\sigma^{\nu\mu} - e\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle fa^*\rangle(q) \\
& +e\langle fa^*\rangle(p)\langle\lambda\bar{\lambda}\rangle(p-q)\frac{\not{p}-\not{q}}{(p-q)^2} - ie\langle aa^*\rangle(p)\langle\lambda\bar{\lambda}\rangle(p-q).
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
0 = & \ -C\langle b^*(q)\bar{\lambda}(p-q)\psi(p)\rangle^T C^{-1} \not{p}\gamma_5 - \langle\bar{\psi}(q)\lambda(p-q)g(p)\rangle\gamma_5 \\
& -i(p-q)\langle b^*(q)A_\nu(p-q)g(p)\rangle\sigma^{\nu\mu} - e\langle\lambda\bar{\lambda}\rangle\frac{(\not{p}-\not{q})}{(p-q)^2}\langle gb^*\rangle(q) \\
& +e\langle gb^*\rangle(p)\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}. - e\langle bb^*\rangle(p)\langle\lambda\bar{\lambda}\rangle(p-q).
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
0 = & \ +iC\langle f^*(q)\bar{\lambda}(p-q)\psi(p)\rangle^T C^{-1} - e\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle af^*\rangle(q) \\
& -i(p-q)_\mu\langle f^*A_\nu a\rangle\sigma^{\nu\mu} - i\langle\bar{\psi}\lambda b\rangle \not{q} \\
& +e\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle af^*\rangle(p) \\
& -e\langle\lambda\bar{\lambda}\rangle(p-q)\langle bb^*\rangle(p).
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
0 = & \ -C\langle g^*(q)\bar{\lambda}(p-q)\psi(p)\rangle^T C^{-1}\gamma_5 - e\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle bg^*\rangle(q) \\
& -i(p-q)_\mu\langle g^*A_\nu b\rangle\sigma^{\nu\mu} + \langle\bar{\psi}\lambda b\rangle \not{q}\gamma_5 \\
& +e\langle\lambda\bar{\lambda}\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle bg^*\rangle(p) \\
& +e\langle\lambda\bar{\lambda}\rangle(p-q)\langle bb^*\rangle(p).
\end{aligned}
\tag{A.6}
$$

$$0 = -iC\langle f^*(q)\bar\lambda(p-q)\psi(p)\rangle^T C^{-1} \not{p} + i\langle\bar\psi(q)\lambda(p-q)f(p)\rangle$$
$$-i(p-q)_\mu\langle f^*(q)A_\nu(p-q)f(p)\rangle\sigma^{\nu\mu} - e\langle\lambda\bar\lambda\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle ff^*\rangle(q)$$
$$+e\langle ff^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2} + ie\langle fa^*\rangle(p)\langle\lambda\bar\lambda\rangle\frac{(\not{p}-\not{q})}{(p-q)^2}$$
$$-ie\langle af^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}. \tag{A.7}$$

$$0 = -C\langle g^*(q)\bar\lambda(p-q)\psi(p)\rangle^T C^{-1} \not{p}\gamma_5 + \langle\bar\psi(q)\lambda(p-q)g(p)\rangle \not{q}\gamma_5$$
$$-i(p-q)_\mu\langle g^*(q)A_\nu(p-q)g(p)\rangle\sigma^{\nu\mu} - e\langle\lambda\bar\lambda\rangle(p-q)\frac{(\not{p}-\not{q})}{(p-q)^2}\langle gg^*\rangle(q)$$
$$+e\langle gg^*\rangle(p)\langle\lambda\bar\lambda\rangle\frac{(\not{p}-\not{q})}{(p-q)^2} - e\langle bg^*\rangle(q)\langle\lambda\bar\lambda\rangle(p-q)$$
$$+e\langle bg^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q). \tag{A.8}$$

$$0 = -i\,\not{p}\langle\bar\psi(q)A_\mu\psi(p)\rangle - ie\langle\psi\bar\psi\rangle(q)\gamma^\nu\langle A_\nu A_\mu\rangle(p-q)$$
$$+\left(\gamma_\mu - \frac{(p-q)_\mu}{\not{p}-\not{q}}\right)\langle\bar\psi(q)\bar\lambda(p-q)f(p)\rangle + \langle f^*(q)A_\mu(p-q)f(p)\rangle$$
$$- \not{q}\langle a^*(q)A_\mu(p-q)f(p)\rangle + e\langle A_\mu A_\nu\rangle(p-q)\gamma^\nu\langle fa^*\rangle(p) \tag{A.9}$$

$$0 = \gamma_5\,\not{p}\langle\bar\psi(q)A_\mu(p-q)\psi(p)\rangle + e\gamma_5\langle\psi\bar\psi\rangle(q)\gamma^\nu\langle A_\nu A_\mu\rangle(p-q)$$
$$-\left(\gamma_\mu - \frac{(p-q)_\mu}{\not{p}-\not{q}}\right)\langle\bar\psi(q)\bar\lambda(p-q)g(p)\rangle + i\gamma_5\langle g^*(q)A_\mu(p-q)g(p)\rangle$$
$$-i\gamma_5\,\not{q}\langle b^*(q)A_\mu(p-q)g(p)\rangle + e\langle A_\mu A_\nu\rangle(p-q)\gamma_5\gamma^\nu\langle gb^*\rangle(q) \tag{A.10}$$

$$0 = i\langle\bar\psi(q)A_\mu(p-q)\psi(p)\rangle - \left(\gamma_\mu - \frac{(p-q)_\mu}{\not{p}-\not{q}}\right)\langle\bar\psi(q)\lambda(p-q)a(p)\rangle$$
$$+\langle f^*(q)A_\mu(p-q)a(p)\rangle - \not{q}\langle a^*(q)A_\mu(p-q)a(p)\rangle$$
$$+e\langle A_\mu A_\nu\rangle(p-q)\gamma^\nu\langle aa^*\rangle(p) \tag{A.11}$$

$$0 = -\gamma_5\langle\bar\psi(q)A_\mu(p-q)\psi(p)\rangle - \left(\gamma_\mu - \frac{(p-q)_\mu}{\not{p}-\not{q}}\right)\langle\bar\psi(q)\lambda(p-q)b(p)\rangle$$
$$+i\gamma_5\langle g^*(q)A_\mu(p-q)b(p)\rangle - i\gamma_5\,\not{q}\langle b^*(q)A_\mu(p-q)b(p)\rangle$$
$$-ie\gamma_5\langle A_\mu A_\nu\rangle(p-q)\gamma^\nu\langle bb^*\rangle(p) \tag{A.12}$$

$$0 = -\langle f^*(q)\bar\lambda(p-q)\psi(p)\rangle_\alpha{}^\gamma \delta_\kappa{}^\beta - i\langle g^*(q)\bar\lambda(p-q)\psi(p)\rangle_\alpha{}^\gamma (\gamma_5)_\kappa{}^\beta$$
$$+\langle a^*(q)\bar\lambda(p-q)\psi(p)\rangle_\alpha{}^\gamma (\not{q})_\sigma{}^\beta - i\langle b^*(q)\bar\lambda(p-q)\psi(p)\rangle_\alpha{}^\gamma (\not{q}\gamma_5)_\kappa{}^\beta + ie\langle\psi\bar\psi\rangle_\alpha{}^\beta(p)\delta_\kappa{}^\gamma$$
$$-ie\langle\psi\bar\psi\rangle_\alpha{}^\beta(q)\delta_\kappa{}^\gamma - (C)_{\kappa\alpha}(\langle\bar\psi(q)\lambda(p-q)f(p)\rangle^T C^{-1})^{\beta\gamma}$$

$$-i(\gamma_5 C)_{\kappa\alpha}(\langle\bar\psi(q)\lambda(p-q)g(p)\rangle^T C^{-1})^{\beta\gamma} - (\slashed{p}C)_{\kappa\alpha}\langle\bar\psi(q)\lambda(p-q)a(p)\rangle^T (C^{-1})^{\beta\gamma}$$
$$-i(\gamma_5\,\slashed{p}C)_{\kappa\alpha}(\langle b^*(q)\bar\lambda(p-q)\psi(p)\rangle^T C^{-1})^{\beta\gamma} - i\langle\bar\psi(q)A_\nu(p-q)\psi(p)\rangle_\alpha^{\ \beta}(\sigma^{\nu\mu}(p-q)_\mu)_\kappa^{\ \gamma}$$
$$+i\langle\bar\psi(q)D(p-q)\psi(p)\rangle_\alpha^{\ \beta}(\gamma_5)_\kappa^{\ \gamma}. \tag{A.13}$$

$$0 = \langle\bar\psi(q)D(p-q)\psi(p)\rangle + \gamma_5\langle g^*(q)D(p-q)a(p)\rangle$$
$$-\slashed{q}\langle b^*(q)D(p-q)a(p)\rangle + i\gamma_5(\slashed{p}-\slashed{q})\langle\bar\psi(q)\lambda(p-q)a(p)\rangle \tag{A.14}$$

$$0 = \gamma_5\langle\bar\psi(q)D(p-q)\psi(p)\rangle - \langle f^*(q)D(p-q)b(p)\rangle$$
$$+\slashed{q}\langle b^*(q)D(p-q)a(p)\rangle + \gamma_5(\slashed{p}-\slashed{q})\langle\bar\psi*(q)\lambda(p-q)b(p)\rangle \tag{A.15}$$

$$0 = -i\,\slashed{p}\langle\bar\psi(q)D(p-q)\psi(p)\rangle + i\gamma_5\langle g^*(q)D(p-q)f(p)\rangle$$
$$-i\gamma_5\,\slashed{q}\langle b^*(q)D(p-q)f(p)\rangle - \gamma_5(\slashed{p}-\slashed{q})\langle\bar\psi(q)\lambda(p-q)f(p)\rangle \tag{A.16}$$

$$0 = \gamma_5\,\slashed{p}\langle\bar\psi(q)D(p-q)\psi(p)\rangle + \langle f^*(q)D(p-q)g(p)\rangle$$
$$-\slashed{q}\langle a^*(q)D(p-q)g(p)\rangle - \gamma_5(\slashed{p}-\slashed{q})\langle\bar\psi(q)\lambda(p-q)g(p)\rangle \tag{A.17}$$

$$0 = i\langle\bar\psi(q)\lambda(p-q)b(p)\rangle + i\langle a^*(q)D(p-q)b(p)\rangle\gamma_5$$
$$-C\langle a^*(q)\bar\lambda(p-q)\psi(p)\rangle^T C^{-1}\gamma_5 \tag{A.18}$$

$$0 = iC\langle b^*(q)\bar\lambda(p-q)\psi(p)\rangle^T C^{-1} + i\langle b^*(q)D(p-q)a(p)\rangle\gamma_5$$
$$-\langle\bar\psi(q)\lambda(p-q)a(p)\rangle\gamma_5 \tag{A.19}$$

$$0 = -ibar\,\slashed{p}\langle a^*(q)\bar\lambda(p-q)\psi(p)\rangle + ie\gamma_5\langle\lambda\bar\lambda\rangle(p-q)\langle aa^*\rangle(q)$$
$$+i\gamma_5\langle a^*(q)D(p-q)g(p)\rangle + iC^{-1}\langle\bar\psi(q)\lambda(p-q)g(p)\rangle^T C \tag{A.20}$$

$$0 = -i\,\slashed{p}\langle b^*(q)\bar\lambda(p-q)\psi(p)\rangle + ie\gamma_5\langle\lambda\bar\lambda\rangle(p-q)\langle bb^*\rangle(q) + i\gamma_5\langle b^*(q)D(p-q)f(p)\rangle$$
$$-\gamma_5 C\langle\bar\psi(q)\lambda(p-q)f(p)\rangle^T C^{-1} \tag{A.21}$$

$$0 = i\langle g^*(q)\bar\lambda(p-q)\psi(p)\rangle + i\gamma_5\langle g^*(q)D(p-q)a(p)\rangle - ie\langle aa^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q)$$
$$-\gamma_5\,\slashed{q}C\langle\bar\psi(q)\lambda(p-q)a(p)\rangle^T C^{-1} \tag{A.22}$$

$$0 = +i\,\slashed{q}C\langle\bar\psi(q)\lambda(p-q)b(p)\rangle^T C^{-1} - \gamma_5\langle f^*(q)\bar\lambda(p-q)\psi(p)\rangle$$
$$-ie\gamma_5\langle bb^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q) + i\gamma_5\langle f^*(q)D(p-q)b(p)\rangle \tag{A.23}$$

$$0 = i\,\slashed{p}\langle f^*(q)\bar\lambda(p-q)\psi(p)\rangle + e\langle gb^*\rangle(p)\langle\lambda\bar\lambda\rangle$$
$$-\langle f^*(q)D(p-q)g(p)\rangle - \gamma_5\,\slashed{q}C\langle\bar\psi(q)\lambda(p-q)g(p)\rangle^T C^{-1}$$

$$-e\langle af^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q) \tag{A.24}$$

$$
\begin{aligned}
0 = & \; i\gamma_5 \not{p}\langle g^*(q)\bar\lambda(p-q)\psi(p)\rangle + e\langle bg^*\rangle(p)\langle\lambda\bar\lambda\rangle(p-q) \\
& + \langle g^*(q)D(p-q)f(p)\rangle + i \not{q}C\langle\bar\psi(q)\lambda(p-q)f(p)\rangle^T C^{-1} \\
& - e\langle a^* f\rangle(p)\langle\lambda\bar\lambda\rangle(p-q)
\end{aligned} \tag{A.25}
$$

B. A Review of Ward-Takahashi identities

We give here a brief review of the Ward-Takahashi identities These are relations between the vertices and the propagators that follow from gauge symmetry.

Shown in momentum space, the electron-photon vertex is related to the electron propagator by

$$(p-q)\cdot\Gamma_{\bar\psi\psi} = \langle\psi\bar\psi\rangle^{-1}(p) - \langle\psi\bar\psi\rangle^{-1}(q), \tag{B.1}$$

where p, q are the ingoing and outgoing momenta respectively.

Analogous identities hold for the scalar fields. We state them here without further ado.

$$
\begin{aligned}
(p-q)\cdot\Gamma_{aAa}(p,q) &= \langle aa^*\rangle^{-1}(p^2) - \langle aa^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{aAf}(p,q) &= \langle fa^*\rangle^{-1}(p^2) - \langle af^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{fAa}(p,q) &= \langle af^*\rangle^{-1}(p^2) - \langle fa^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{fAf}(p,q) &= \langle ff^*\rangle^{-1}(p^2) - \langle ff^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{bAb}(p,q) &= \langle bb^*\rangle^{-1}(p^2) - \langle bb^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{bAg}(p,q) &= \langle gb^*\rangle^{-1}(p^2) - \langle bg^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{gAb}(p,q) &= \langle bg^*\rangle^{-1}(p^2) - \langle gb^*\rangle^{-1}(q^2), \\
(p-q)\cdot\Gamma_{gAg}(p,q) &= \langle gg^*\rangle^{-1}(p^2) - \langle gg^*\rangle^{-1}(q^2),
\end{aligned} \tag{B.2}
$$

where $\Gamma^\mu_{xAy}(p,q)$ is a photon vertex with an incoming x of momentum p and an outgoing y of momentum q. The photon therefore has momentum $p-q$.

C. Vertex Ward identities

Here we list the vertex Ward identities refered to in Table 2

$$
\begin{aligned}
& \gamma_\mu\Gamma^\mu_{a^*Aa}(p,q) \\
& = \Gamma_{\bar\lambda a^*\psi}(p,q) \not{q} + e(B(p^2) - B(q^2)) + \Gamma_{\bar\lambda a^*\psi}(-q,-p) \not{p},
\end{aligned} \tag{C.1}
$$

$$
\begin{aligned}
& \gamma_\mu\Gamma^\mu_{b^*Ab}(p,q) \\
& = -i\Gamma_{\bar\lambda b^*\psi}(p,q)\gamma_5 \not{q} - e(B(p^2) - B(q^2)) - i\Gamma_{\bar\lambda b^*\psi}(-q,-p)\gamma_5 \not{p},
\end{aligned} \tag{C.2}
$$

$$\gamma_\mu\Gamma^\mu_{f^*Aa}(p,q) + eA(p^2) = \Gamma_{\bar\lambda a^*\psi}(-q,-p) + \Gamma_{\bar\lambda f^*\psi}(p,q) \not{q}, \tag{C.3}$$

$$\gamma_\mu\Gamma^\mu_{g^*Ab}(p,q) - eA(p^2) = i\Gamma_{\bar\lambda b^*\psi}(-q,-p)\gamma_5 + i\Gamma_{\bar\lambda g^*\psi}(p,q) \not{q}\gamma_5, \tag{C.4}$$

$$\gamma_\mu \Gamma^\mu_{f^*Af}(p,q) = \Gamma_{\bar\lambda f^*\psi}(-q,-p) - \Gamma_{\bar\lambda f^*\psi}(p,q), \tag{C.5}$$

$$\gamma_\mu \Gamma^\mu_{g^*Ag}(p,q) = i\Gamma_{\bar\lambda g^*\psi}(-q,-p)\gamma_5 - i\Gamma_{\bar\lambda g^*\psi}(p,q)\gamma_5, \tag{C.6}$$

$$i\sigma^{\mu\nu}(p-q)_\nu \Gamma_{\bar\lambda f^*\psi}(p,q)$$
$$= \Gamma^\mu_{\bar\psi A\psi}(p,q) - i\not{q}\Gamma^\mu_{f^*Af}(p,q) + i\Gamma^\mu_{f^*Aa}(p,q) - ie\gamma^\mu A(p^2), \tag{C.7}$$

$$i\sigma^{\mu\nu}(p-q)_\nu \Gamma_{\bar\lambda g^*\psi}(p,q)$$
$$= i\gamma_5\Gamma^\mu_{\bar\psi A\psi}(p,q) + \gamma_5\not{q}\Gamma^\mu_{g^*Ag}(p,q) - \gamma_5\Gamma^\mu_{g^*Ab}(p,q) + e\gamma_5\gamma^\mu A(p^2), \tag{C.8}$$

$$i\sigma^{\mu\nu}(p-q)_\nu \Gamma_{\bar\lambda a^*\psi}(p,q)$$
$$= i\Gamma^\mu_{a^*Aa}(p,q) - i\not{q}\Gamma^\mu_{a^*Af}(p,q) - e\gamma^\mu\langle\psi\bar\psi\rangle^{-1}(q) - \not{p}\Gamma^\mu_{\bar\psi A\psi}(p,q)$$
$$+ie\gamma^\mu B(p^2), \tag{C.9}$$

$$i\sigma^{\mu\nu}(p-q)_\nu \Gamma_{\bar\lambda b^*\psi}(p,q)$$
$$= -\gamma_5\Gamma^\mu_{b^*Ab}(p,q) + \gamma_5\not{q}\Gamma^\mu_{b^*Ag}(p,q) - i\gamma_5 e\gamma^\mu\langle\psi\bar\psi\rangle^{-1}(q)$$
$$-i\gamma_5\not{p}\Gamma^\mu_{\bar\psi A\psi}(p,q) - e\gamma_5\gamma^\mu B(p^2). \tag{C.10}$$

These last two equations correspond to equations (4.9,4.10) in [14], which contain a typo. Specifically, the argument of the inverse electron propagator is given as p rather than q, as we have done here.

$$0 = -i(\not{q})_\kappa{}^\beta(\Gamma_{\bar\psi f\lambda}(p,q))_\alpha{}^\gamma + (\gamma_5\not{q})_\kappa{}^\beta(\Gamma_{\bar\psi g\lambda}(p,q))_\alpha{}^\gamma$$
$$-i(\not{p}C)_{\alpha\kappa}(C^{-1}\Gamma_{f^*\bar\lambda\psi}(p,q))^{\gamma\beta} + (\not{p}\gamma_5 C)_{\alpha\kappa}(C^{-1}\Gamma_{g^*\bar\lambda\psi}(p,q))^{\gamma\beta}$$
$$+i\delta_\kappa{}^\beta(\Gamma_{\bar\psi a\lambda}(p,q))_\alpha{}^\gamma - (\gamma_5)_\kappa{}^\beta(\Gamma_{\bar\psi b\lambda}(p,q))_\alpha{}^\gamma$$
$$+iC_{\alpha\kappa}(C^{-1}\Gamma_{a^*\bar\lambda\psi}(p,q))^{\gamma\beta} - (\gamma_5 C)_{\alpha\kappa}(C^{-1}_{b^*\bar\lambda\psi}(p,q))^{\gamma\beta}$$
$$+(\gamma_\nu)_\kappa{}^\gamma(\Gamma^\nu_{\bar\psi A\psi}(p,q))_\alpha{}^\beta - (\gamma_5(\not{p}-\not{q}))_\kappa{}^\gamma(\Gamma_{\bar\psi D\psi}(p,q))_\alpha{}^\beta, \tag{C.11}$$

where C is the charge conjugation matrix. Note that derivation of this idientity requires the Ward-Takahashi identity (B.1). We obtain

$$0 = (\not{p}-\not{q})\gamma_5\mathrm{Tr}(\Gamma_{\bar\psi D\psi}(p,q)) + \gamma_\mu\mathrm{Tr}(\Gamma^\mu_{\bar\psi A}(p,q)) + i\Gamma_{\bar\psi a\lambda}(p,q)$$
$$-\gamma_5\Gamma_{\bar\psi b\lambda}(p,q) - i\Gamma_{\bar\psi a\lambda\psi}(-q,-p) + \gamma_5\Gamma_{\bar\psi b\lambda}(-q,-p)$$
$$-i\not{q}\Gamma_{\bar\psi f\lambda}(p,q) + \gamma_5\not{q}\Gamma_{\bar\psi g\lambda}(p,q) - i\not{p}\Gamma_{\bar\psi f\lambda}(-q,-p)$$
$$+\gamma_5\not{p}\Gamma_{\bar\psi g\lambda}(-q,-p), \tag{C.12}$$

by setting $\beta = \alpha$ and summing, and

$$0 = i\mathrm{Tr}(\Gamma_{\bar\psi a\lambda}(p,q)) - \gamma_5\mathrm{Tr}(\Gamma_{\bar\psi b\lambda}(p,q)) - i\not{q}\mathrm{Tr}(\Gamma_{\bar\psi f\lambda}(p,q))$$
$$+\gamma_5\not{q}\mathrm{Tr}(\Gamma_{\bar\psi g\lambda}(p,q)) - i\Gamma_{\bar\lambda a^*\psi}(p,q) + \gamma_5\Gamma_{\bar\lambda b^*\psi}(p,q) - i\not{p}\Gamma_{\bar\lambda f^*\psi}(p,q)$$
$$-\not{p}\gamma_5\Gamma_{\bar\lambda g^*\psi}(p,q) + \gamma_\mu\Gamma^\mu_{\bar\psi A\psi}(p,q) - \gamma_5(\not{p}-\not{q})\Gamma_{\bar\psi D\psi}(p,q), \tag{C.13}$$

by setting $\gamma = \alpha$ and summing.

$$i\gamma_5\Gamma_{\bar\lambda a^*\psi}(p,q)$$
$$= \not p\Gamma_{\bar\psi D\psi}(p,q) + \gamma_5\Gamma_{a^*Db}(p,q) - \gamma_5\not q\Gamma_{a^*Dg}(p,q), \tag{C.14}$$

$$i\gamma_5\Gamma_{\bar\lambda b^*\psi}(p,q)$$
$$= i\gamma_5\not p\Gamma_{\bar\psi D\psi}(p,q) - i\Gamma_{b^*Da}(p,q) + i\not q\Gamma_{b^*Df}(p,q), \tag{C.15}$$

$$\gamma_5\Gamma_{f^*Db}(p,q)$$
$$= i\gamma_5\Gamma_{\bar\lambda f^*\psi}(p,q) + \gamma_5\not q\Gamma_{f^*Dg}(p,q) + \Gamma_{\bar\psi D\psi}(p,q), \tag{C.16}$$

$$\gamma_5\Gamma_{g^*Da}(p,q)$$
$$= -\Gamma_{\bar\lambda g^*\psi}(p,q) + \gamma_5\not q\Gamma_{g^*Df}(p,q) - \Gamma_{\bar\psi D\psi}(p,q), \tag{C.17}$$

$$\gamma_5(\not p - \not q)\Gamma_{a^*Db}(p,q)$$
$$= \Gamma_{\bar\lambda b^*\psi}(-q,-p)\not p + i\Gamma_{\bar\lambda a^*\psi}(p,q)\not q\gamma_5 + ie\gamma_5(B(p^2) - B(q^2)), \tag{C.18}$$

$$\gamma_5(\not p - \not q)\Gamma_{b^*Da}(p,q)$$
$$= i\Gamma_{\bar\lambda a^*\psi}(-q,-p)\not p\gamma_5 + \Gamma_{\bar\lambda b^*\psi}(p,q)\not q + ie\gamma_5(B(p^2) - B(q^2)), \tag{C.19}$$

$$\gamma_5(\not p - \not q)\Gamma_{a^*Dg}(p,q)$$
$$= \Gamma_{\bar\lambda g^*\psi}(-q,-p)\not p - i\Gamma_{\bar\lambda a^*\psi}(p,q)\gamma_5 + ie\gamma_5 A(q^2), \tag{C.20}$$

$$\gamma_5(\not p - \not q)\Gamma_{b^*Df}(p,q)$$
$$= i\Gamma_{\bar\lambda f^*\psi}(-q,-p)\not p\gamma_5 - \Gamma_{\bar\lambda b^*\psi}(p,q) + ie\gamma_5 A(q^2), \tag{C.21}$$

$$\gamma_5(\not p - \not q)\Gamma_{g^*Da}(p,q)$$
$$= \Gamma_{\bar\lambda g^*\psi}(p,q)\not q + i\Gamma_{\bar\lambda a^*\psi}(-q,-p)\gamma_5 - ie\gamma_5 A(p^2), \tag{C.22}$$

$$\gamma_5(\not p - \not q)\Gamma_{f^*Db}(p,q)$$
$$= i\Gamma_{\bar\lambda f^*\psi}(p,q)\not q\gamma_5 + \Gamma_{\bar\lambda b^*\psi}(-q,-p) - ie\gamma_5 A(p^2), \tag{C.23}$$

$$\gamma_5(\not p - \not q)\Gamma_{f^*Dg}(p,q) = \Gamma_{\bar\lambda g^*\psi}(-q,-p) - i\Gamma_{\bar\lambda f^*\psi}(p,q)\gamma_5, \tag{C.24}$$

$$\gamma_5(\not p - \not q)\Gamma_{g^*Df}(p,q) = i\Gamma_{\bar\lambda f^*\psi}(-q,-p)\gamma_5 - \Gamma_{\bar\lambda g^*\psi}(p,q). \tag{C.25}$$

D. General Solution of SUSY Vertex Ward identities

Below is a solution for the SWIs of appendix C and WTIs of appendix B. It is the most general set of vertices consistent with both sets of identities and free of kinematic singularities if one assumes charge conjugation invariance and

$$\Gamma^\mu_{a^*Aa}(p,q) = \Gamma^\mu_{b^*Ab}(p,q),$$
$$\Gamma^\mu_{f^*Aa}(p,q) = \Gamma^\mu_{g^*Ab}(p,q),$$
$$\Gamma^\mu_{a^*Af}(p,q) = \Gamma^\mu_{b^*Ag}(p,q),$$
$$\Gamma^\mu_{f^*Af}(p,q) = \Gamma^\mu_{g^*Ag}(p,q), \tag{D.1}$$

The proof of this is presented in [14]. The assumption of Eq. (D.1) is true to all orders in perturbation theory, and any nonperturbative violations of this assumption are restricted by the WTIs to lie completely within their transverse components. (Note that two vertices contained typographical errors in the original paper [14]. These are noted in the body of the text. We have checked that the proof of their uniqueness is still valid.) Our general solution is as follows:

The scalar-photon vertices are

$$
\Gamma^\mu_{a^*Aa}(p,q) = \Gamma^\mu_{b^*Ab}(p,q) = \frac{e}{p^2-q^2}(p^2A(p^2)-q^2A(q^2))(p+q)^\mu
$$
$$
+[p^\mu(q^2-p\cdot q)+q^\mu(p^2-p\cdot q)]T_{aa}(p^2,q^2,p\cdot q), \tag{D.2}
$$

$$
\Gamma^\mu_{a^*Af}(p,q) = \Gamma^\mu_{b^*Ag}(p,q) = \Gamma^\mu_{f^*Aa}(p,q) = \Gamma^\mu_{g^*Ab}(p,q)
$$
$$
= \frac{-e}{p^2-q^2}(B(p^2)-B(q^2))(p+q)^\mu
$$
$$
+[p^\mu(q^2-p\cdot q)+q^\mu(p^2-p\cdot q)]T_{af}(p^2,q^2,p\cdot q), \tag{D.3}
$$

$$
\Gamma^\mu_{f^*Af}(p,q) = \Gamma^\mu_{g^*Ag}(p,q) = \frac{e}{p^2-q^2}(A(p^2)-A(q^2))(p+q)^\mu
$$
$$
+[p^\mu(q^2-p\cdot q)+q^\mu(p^2-p\cdot q)]T_{ff}(p^2,q^2,p\cdot q), \tag{D.4}
$$

where the three functions $T_{aa}(p^2,q^2,p{\cdot}q)$, $T_{af}(p^2,q^2,p{\cdot}q)$ and $T_{ff}(p^2,q^2,p{\cdot}q)$, each satisfying $T(p^2,q^2,p\cdot q)=T(q^2,p^2,p\cdot q)$, are free of kinematic singularities and represent the only degrees of freedom inherent in the solution. The forms (D.2) to (D.4) are equivalent to that given by Ball and Chiu [2] in the context of non-SUSY scalar QED. The photino vertices are

$$
\Gamma_{\bar\lambda a^*\psi}(p,q) = \frac{e}{p^2-q^2}(p^2A(p^2)-q^2A(q^2)) + \frac{e}{p^2-q^2}(B(p^2)-B(q^2))\,\slashed{q}
$$
$$
+\frac{1}{2}e(p^2- \slashed{q}\,\slashed{p})T_{aa}(p^2,q^2,p\cdot q)
$$
$$
+\frac{1}{2}e[\slashed{p}(p^2-q^2)-2\,\slashed{q}(p^2-p\cdot q)]T_{af}(p^2,q^2,p\cdot q),
$$
$$
+\frac{1}{2}ep^2(q^2- \slashed{p}\,\slashed{q})T_{ff}(p^2,q^2,p\cdot q), \tag{D.5}
$$

and

$$
\Gamma_{\bar\lambda f^*\psi}(p,q) = \frac{-e}{p^2-q^2}(A(p^2)-A(q^2))\,\slashed{q} - \frac{e}{p^2-q^2}(B(p^2)-B(q^2))
$$
$$
+\frac{1}{2}e(\slashed{p}- \slashed{q})T_{aa}(p^2,q^2,p\cdot q)
$$
$$
+\frac{1}{2}e(p-q)^2T_{af}(p^2,q^2,p\cdot q)
$$
$$
-\frac{1}{2}e\,\slashed{q}(p^2- \slashed{p}\,\slashed{q})T_{ff}(p^2,q^2,p\cdot q). \tag{D.6}
$$

The vertices of b,g with the photino are

$$
\Gamma_{\bar\lambda b^*\psi}(p,q) = i\gamma_5\Gamma_{\bar\lambda a^*\psi}(p,q), \tag{D.7}
$$

$$\Gamma_{\bar{\lambda}g^*\psi}(p,q) = i\gamma_5\Gamma_{\bar{\lambda}f^*\psi}(p,q). \tag{D.8}$$

The electron-photon vertex must be restricted at least to the form given by Ball and Chiu [2] for non-SUSY QED. For the SUSY case we find

$$\begin{aligned}
\Gamma^\mu_{\bar{\psi}A\psi}(p,q) &= \Gamma^\mu_{BC}(p,q) + \frac{ie}{p^2-q^2}(A(p^2)-A(q^2))[\frac{1}{2}T_3^\mu - T_8^\mu] \\
&\quad -\frac{ie}{p^2-q^2}(B(p^2)-B(q^2))T_5^\mu + \frac{1}{2}ieT_{aa}(p^2,q^2,p\cdot q)T_3^\mu \\
&\quad +ieT_{af}(p^2,q^2,p\cdot q)[\frac{1}{2}(p-q)^2T_5^\mu - T_1^\mu] \\
&\quad +\frac{1}{2}ieT_{ff}(p^2,q^2,p\cdot q)[T_2^\mu - p\cdot qT_3^\mu - (p-q)^2T_8^\mu], \tag{D.9}
\end{aligned}$$

where

$$\begin{aligned}
\Gamma^\mu_{BC}(p,q) &= \frac{1}{2}\frac{ie}{p^2-q^2}(\slashed{p}+\slashed{q})(A(p^2)-A(q^2))(p+q)^\mu \\
&\quad +ie\frac{1}{2}(A(p^2)+A(q^2))\gamma^\mu + \frac{ie}{p^2-q^2}(B(p^2)-B(q^2))(p+q)^\mu, \tag{D.10}
\end{aligned}$$

$$T_1^\mu = p^\mu(q^2-p\cdot q) + q^\mu(p^2-p\cdot q), \tag{D.11}$$
$$T_2^\mu = (\slashed{p}+\slashed{q})T_1^\mu, \tag{D.12}$$
$$T_3^\mu = \gamma^\mu(p-q)^2 - (\slashed{p}-\slashed{q})(p-q)^\mu], \tag{D.13}$$
$$T_5^\mu = \sigma^{\mu\nu}(p-q)_\nu, \tag{D.14}$$
$$T_8^\mu = \frac{1}{2}(\slashed{p}\,\slashed{q}\gamma^\mu - \gamma^\mu\,\slashed{q}\,\slashed{p}). \tag{D.15}$$

Finally there are the vertices for the D-boson, namely,

$$\begin{aligned}
\Gamma_{a^*Db}(p,q) &= -\Gamma_{b^*Da}(p,q) \\
&= \frac{ie}{p^2-q^2}(p^2A(p^2)-q^2A(q^2)) - iep\cdot qT_{a^*a}(p^2,q^2,p\cdot q) \\
&\quad +iep^2q^2T_{ff}(p^2,q^2,p\cdot q), \tag{D.16}
\end{aligned}$$

$$\begin{aligned}
\Gamma_{f^*Dg}(p,q) &= -\Gamma_{g^*Df}(p,q) \\
&= \frac{ie}{p^2-q^2}(A(p^2)-A(q^2)) + ieT_{a^*a}(p^2,q^2,p\cdot q) \\
&\quad -iep\cdot qT_{f^*f}(p^2,q^2,p\cdot q), \tag{D.17}
\end{aligned}$$

$$\begin{aligned}
\Gamma_{g^*Da}(p,q) &= \frac{ie}{p^2-q^2}(B(p^2)-B(q^2)) \\
\Gamma_{g^*Da}(p,q) &= \frac{ie}{p^2-q^2}(B(p^2)-B(q^2)) \\
&\quad -ie(q^2-p\cdot q)T_{af}(p^2,q^2,p\cdot q), \tag{D.18}
\end{aligned}$$

$$\Gamma_{a^*Dg}(p,q) = \frac{-ie}{p^2-q^2}(B(p^2)-B(q^2))$$
$$+ie(p^2-p\cdot q)T_{af}(p^2,q^2,p\cdot q), \qquad (D.19)$$

$$\Gamma_{f^*Db}(p,q) = \frac{-ie}{p^2-q^2}(B(p^2)-B(q^2))$$
$$+ie(q^2-p\cdot q)T_{af}(p^2,q^2,p\cdot q), \qquad (D.20)$$

$$\Gamma_{b^*Df}(p,q) = \frac{ie}{p^2-q^2}(B(p^2)-B(q^2))$$
$$-ie(p^2-p\cdot q)T_{af}(p^2,q^2,p\cdot q), \qquad (D.21)$$

and

$$\Gamma_{\bar{\psi}D\psi}(p,q) = -\frac{1}{2}ie\gamma_5[(\not{p}+\not{q})T_{a^*a}(p^2,q^2,p\cdot q)$$
$$-(p^2-q^2)T_{af}(p^2,q^2,p\cdot q)$$
$$+(\not{q}p^2+\not{p}q^2)T_{ff}(p^2,q^2,p\cdot q)]. \qquad (D.22)$$

The first of these vertices has an erroneous factor of half, and the last vertex has an overall sign error, in [14].

References

[1] J.C. Ward, *An identity in quantum electrodynamics*, Phys. Rev. **78** *(1950) 182*.

[2] J.S. Ball and T.W. Chiu, *Analytic behaviour of vertex functions in gauge theories. i*, Phys. Rev. **D 22** *(1980) 2542*.

[3] J. Iliopoulos and B. Zumino, *Broken supergauge symmetry and renormalization*, Nucl. Phys. **B 76** *(1974) 310*.

[4] O. Piguet and K. Sibold, *Renormalised Supersymmetry*. Birkhäuser, Boston, (1986).

[5] C. Rupp, R. Scharf, and K. Sibold, *Supersymmetry transformation of quantum fields*, Nucl. Phys. **B 612** *(2001) 313-319* [`hep-th/0101153`].

[6] T.Ohl and J. Reuter, *Clockwork susy: Supersymmetric ward and slavnov-taylor identities at work in green's functions and scattering amplitudes*, Eur. Phys. J. **C 30** *(2003) 525* **C30** (2003) 525, [`hep-th/0212224`].

[7] A. Soloshenko and K. Stepanyantz, *Two-loop renormalization of n = 1 supersymmetric electrodynamics, regularized by higher derivatives*, `hep-th/0203118`.

[8] A. A. Slavnov and K. V. Stepanyantz, *Universal invariant renormalization for supersymmetric theories*, Theor. Math. Phys. **135** *(2003) 673-684* [`hep-th/0208006`].

[9] A. A. Slavnov and K. V. Stepanyantz, *Universal invariant renormalization of supersymmetric yang- mills theory*, Theor. Math. Phys. **139** *(2004) 599-608* [`hep-th/0305128`].

[10] J. Wess and B. Zumino, *Supergauge invariant extension of quantum electrodynamics*, Nucl. Phys. **B 78** *(1974) 1*.

[11] M. L. Walker, *A completely invariant susy transform of supersymemtric qed*, J. Phys. **G 30** *(2004) 1219-1224* [`hep-th/0310098`].

[12] S. Weinberg, *The Quantum Theory of Fields II*. Cambridge University Press, New York, (1996).

[13] V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories*. World Scientific, Singapore, (1993).

[14] M. L. Walker and C. J. Burden, *Nonperturbative vertices in supersymmetric quantum electrodynamics*, Phys. Rev. **D 60** *(1999) 105018* [`hep-th/9904144`].

[15] M. Koopmans and J. J. Steringa, *Dynamical mass generation in supersymmetric qed in three-dimensions*, Phys. Lett. **B 226** *(1989) 309*.

[16] P. Ramond, *Field Theory: A Modern Primer*. Benjamin-Cummings, Massachusetts, (1981).